Erratum to "Exact Algorithms for the Clustered Vehicle Routing Problem "

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The Clustered Vehicle Routing Problem (CluVRP) is a generalization of the Capacitated Vehicle Routing Problem (CVRP) were customers are partitioned into disjoint clusters. As in the CVRP, all the customers must be visited exactly once, but a vehicle visiting one customer in a cluster must visit all the remaining customers therein before leaving it. Hence, when the first and last customer to be visited in a cluster C are p and q, finding the optimal sequence to visit the remaining customers in the cluster is equal to finding the minimum cost Hamiltonian path with p and q as endpoints.
Battarra et al. [1] presented a new compact integer programming formulation for the CluVRP. This formulation exploits the special substructure of the clusters such that only inter cluster connections have to be determined. The authors show that this formulation provides a stronger linear relaxation than the CluVRP formulation based on the tradional CVRP two-index formulation.
In the sequence of propositions that show the equivalence of the new formulation with the traditional CluVRP formulation an erroneous notation was introduced in [1]. In particular, this appears in the proof of Lemma 2, which is rewritten here in correct form. We emphasize that the incorrect notation in the proof does not affect the overall validity of Lemma 2, hence has no effect on the correctness of the paper and on the results reported therein.
Let $\delta(S)$ denote the set of edges connecting the vertices in a vertex set $S \subseteq V$ to those outside the set, and the set of edges inside a vertex set S by $E(S)$. The decision variables x^*_{ij} and x^{**}_{ij} are equal to the number of times a vehicle traverses edge (i, j) for a solution of the traditional and the new formulation, respectively. The values of x^*_{ij} are determined through the following transformation from x^{**}_{ij}. For each inter-cluster edge $(i, j) \in \bar{E}$, set $x^*_{ij} = x^{**}_{ij}$. Initialize $x^*_{ij} = 0$ for all intra-cluster edges $(i, j) \in \hat{E}$. For all $(p, q) \in \hat{E}$ and for all $(i, j) \in P(p, q)$, increment x^*_{ij} by x^{**}_{pq}. For the details of the CluVRP formulations, we refer the reader to Battarra et al. [1].

Lemma 2. *Given a set of customer vertices $S \subset V \backslash \{0\}$, with $\bar{C}$ being the minimal set of clusters covering S, i.e. $\forall i \in S, \exists C \in \bar{C} : i \in C$, the inequality $\sum_{(i,j)\in\delta(S)} x^*_{ij} \geq \sum_{(i,j)\in\delta(\bar{C})} x^*_{ij}$ holds.*

Proof. By induction on $|\bar{C}|$.
Base case: For $|\bar{C}| = 1$, let $\bar{C} = \{C\}$. For $|C| \leq 2$, the inequality holds as an equality. For $|C| \geq 3$, we have the following:

$$\begin{aligned}
&\sum_{(i,j)\in\delta(S)} x^*_{ij} \\
&= \sum_{(i,j)\in\delta(C)} x^*_{ij} - \sum_{(i,j)\in\delta(C)\backslash\delta(S)} x^*_{ij} + \sum_{(i,j)\in E(C)\bigcap\delta(S)} x^*_{ij} && (23)\\
&= \sum_{(i,j)\in\delta(C)} x^*_{ij} - \sum_{(i,j)\in\delta(C)\backslash\delta(S)} x^{**}_{ij} + \sum_{(i,j)\in E(C)\bigcap\delta(S)} x^*_{ij} && (24)\\
&\geq \sum_{(i,j)\in\delta(C)} x^*_{ij} - \sum_{(i,j)\in\delta(C)\backslash\delta(S)} x^{**}_{ij} + \sum_{(p,q)\in E(C)\bigcap\delta(S)} x^{**}_{pq} + \sum_{(p,q)\in E(C)\backslash\delta(S)} 2x^{**}_{pq} && (25)\\
&\geq \sum_{(i,j)\in\delta(C)} x^*_{ij} - \sum_{(p,q)\in E(C)\backslash(\delta(S)\bigcup E(S))} 2x^{**}_{pq} + \sum_{(p,q)\in E(C)\backslash\delta(S)} 2x^{**}_{ij} && (26)\\
&\geq \sum_{(i,j)\in\delta(C)} x^*_{ij} && (27)
\end{aligned}$$

Equality (23) states the relationship between $\sum_{(i,j)\in\delta(S)} x^*_{ij}$ and $\sum_{(i,j)\in\delta(C)} x^*_{ij}$, where the former is calculated by removing the inter-cluster edges in $\delta(C)\backslash\delta(S)$ from $\delta(C)$ and by adding the intra-cluster edges belonging to $E(C)\bigcap\delta(S)$. Equality (24) follows from the transformation from x^* to x^{**}. Inequality (25) follows from the fact that a Hamiltonian path in C with endpoints p and q will cross between S and $C\backslash S$ at least once if one endpoint lies in S and the other endpoint in $C\backslash S$ (i.e., $(p,q) \in E(C)\bigcap\delta(S)$), and at least twice if both p and q are in S or in $C\backslash S$ (i.e., $(p,q) \in E(C)\backslash\delta(S)$). Inequality (26) is due to the fact that $\sum_{(i,j)\in\delta(C)\backslash\delta(S)} x^{**}_{ij} = \sum_{(p,q)\in E(C)\bigcap\delta(S)} x^{**}_{pq} + \sum_{(p,q)\in E(C)\backslash(\delta(S)\bigcup E(S))} 2x^{**}_{pq}$ an implication of the edge degree constraint (9) in the new formulation. Finally, (27) follows from the fact that $E(C)\backslash(\delta(S)\bigcup E(S)) \subseteq E(C)\backslash\delta(S)$.
The proof is completed by the inductive step which can be found in Battarra et al. [1]. ☐

References

[1] Maria Battarra, Güneş Erdoğan, Daniele Vigo (2014) Exact Algorithms for the Clustered Vehicle Routing Problem. Operations Research 62(1):58-71